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Performance of Renormalization Group Algebraic Turbulence Model on Boundary Layer Transition Simulation

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ABSTRACT

The RNG-based algebraic turbulence model, with a new method of solving the cubic equation and applying new length scales, has been introduced. An analysis has been made of the RNG length scale which was previously reported and the resulting eddy viscosity has been compared with those from other algebraic turbulence models. Subsequently, a new length scale is introduced which actually uses the two previous RNG length scales in a systematic way to improve the model performance. The performance of the present RNG model has been demonstrated by simulating the boundary layer flow over a flat plate and the flow over an airfoil.

NOMENCLATURE

A_d constant ($=0.2$)
b constant ($=\frac{3}{8}A_d\frac{1.594}{\pi^4}$)
c airfoil chord length
C wave length cut-off constant (75-200)
C_f skin friction coefficient
C_μ turbulence model constant ($=0.09$)
F_{Kleb} Klebanoff's intermittency factor
H Heavyside function
l mixing length
l_1 mixing length from Ref. 2
l_2 mixing length from Ref. 4
L_f integral scale of turbulence
Re_c Reynolds number based on chord length
Re_x local Reynolds number
Re_θ Reynolds number based on local momentum thickness
RNG   Renormalization Group
t time
TI free stream turbulent intensity
u stream-wise velocity component
U mean velocity component in streamwise direction
x coordinate direction along chord line
y coordinate normal to the wall

Greek symbols

α outer model constant ($=0.0068$)
β turbulence model constant ($=\frac{\kappa}{C_\mu}$)
δ boundary layer thickness
δ^* displacement thickness
ϵ turbulent eddy viscosity
θ momentum thickness
κ von Karman constant ($=0.4$)
ν_0 kinematic viscosity
ν_t the provisional eddy viscosity
ν effective viscosity
ρ density
σ wall friction parameter ($=\sqrt{\frac{|C_f|}{2}}$)
τ shear stress
Π Coles pressure gradient parameter
Λ_f wave number

Superscripts

$\hat{}$ normalized quantity
$\bar{}$ time averaged quantity
$+$ law-of-the-wall coordinate

Subscripts

0 laminar quantity
i inner layer
o outer layer
t turbulent quantity
e quantity at outer edge of boundary layer
$rng1$ based on length scale from Ref. 2
$rng2$ based on length scale from Ref. 4
bl based on length scale from Ref. 6

1. INTRODUCTION

Studies of the transition from laminar to turbulent flow in the boundary layer of an airfoil were performed in the 1940's and earlier, upon observing that extensive

laminar layers existed on the forward portion of smooth wings. However, in recent years only limited numbers of turbulence models capable of simulating the laminar-turbulent transition have been developed.

The drag of an object such as a flat plate or an airfoil with a smooth surface depends on the location of the point of transition where the value of drag suddenly changes from low-drag laminar-type flow to high-drag turbulent flow. An incorrect estimation of the transition point greatly decreases the level of accuracy in predicting the flow characteristics farther downstream. Therefore, the accurate prediction of drag by CFD methods requires the correct prediction of the transition location.

In 1986, Yakhot and Orszag[1] proposed the application of the renormalization group theory to turbulence modeling for transition simulation. Later Martinelli and Yakhot[2] proposed an algebraic and differential $k - \epsilon$ model based on the RNG theory of Yakhot and Orszag[1] and applied the algebraic RNG model to transonic flows. Lund[3] investigated the algebraic RNG model and used a simple cubic equation for ν after imposing reasonable assumptions. Kirtley[4] used a quartic equation based on the cubic equation and applied the model to three dimensional turbomachinery using Newton's method to find the roots of the quartic equation.

The main reason for using a quartic equation is due to the existence of more than one solution to the cubic equation whose locus of real roots consists of two branches. By special transformation to the quartic equation, a difficulty of finding one non-trivial solution was simply eliminated and Newton's method was used to evaluate the solution to the transformed quartic equation of Kirtley[4]. Therefore, both branches have been retained to avoid a jump discontinuity in the distribution of effective viscosity, and the solution was evaluated according to some forcing conditions given by Lund[3] [Figure 1]. Kirtley[4] successfully used a method stemming from direct evaluation of the cubic equation with the jump condition and showed improvements over the Baldwin-Lomax model with modification for wake and separated flows.

In the present study, one of the solution branches that shows less physically important behavior than the other real solution branch has also been eliminated and the solution has been evaluated according to a general solution formula for a cubic equation which involves algebraic manipulation of complex numbers. The resulting jump discontinuity in normalized viscosity does not show any negative contribution to convergence in the main flow solver [Figure 2]. The present method is compact in size and simple in analysis since it only requires the evaluation of the cubic equation at each mesh point. Due to the straightforwardness of the present analysis, the implementation effort for coding has been minimized. The main intent of the present study is to provide a better understanding of the model's transition behavior and to provide extensive flow results in order to check the performance of the model on boundary layer transition simulation as well as turbulent simulation.

2. DERIVATION OF THE MODEL

The mathematical form of the RNG theory is given by Yakhot and Orszag[1] as

$$\nu = \nu_0 \left[1 + H \left(\frac{3}{8} A_d \frac{1.594}{\nu_0^3 \Lambda_f^4} \bar{\epsilon} - C \right) \right]^{1/3} \qquad (1)$$

where $H(x)$ is a Heavyside function defined as follows:

$$H(x) = \begin{cases} x, & \text{for } x \geq 0; \\ 0, & \text{for } x < 0. \end{cases}$$

In Equation (1), ν and ν_0 are the effective and kinematic viscosity respectively, A_d is a constant (=0.2), and Λ_f is a wave number corresponding to the integral scale of the turbulence in the inertial range. A cut-off constant derived by Yakhot and Orszag[1] is represented by C (=75-200), and $\bar{\epsilon}$ is the dissipation rate.

Lund[3] derived the following cubic RNG formulation from Equation (1) after a series of simplifications:

$$\hat{\nu} = \left[1 + H \left(b \left(\frac{L_f}{l} \right)^4 \hat{\nu} \hat{\nu}_t^2 - C \right) \right]^{1/3} \qquad (2)$$

where L_f is an integral length scale, and ν_t can be determined from Prandtl's mixing length theory:

$$\nu_t = l^2 \mid \frac{\partial \bar{u}}{\partial y} \mid \qquad (3)$$

Also by applying the fully turbulent condition, that is

$$\hat{\nu} \rightarrow \hat{\nu}_t + 1 \simeq \hat{\nu}_t \qquad for \quad \hat{\nu}_t \gg 1 \qquad (4)$$

the integral and mixing length scales ratio can be represented as follows:

$$b\left(\frac{L_f}{l}\right)^4 = 1 + \frac{1}{\hat{\nu}_t^3}(C-1) \approx 1 \qquad (5)$$

which leads to the final governing equation for effective viscosity not only valid in laminar regime but also in the transition/turbulent regime, that is

$$\hat{\nu} = \left[1 + H\left(\hat{\nu}\hat{\nu}_t^2 - C\right)\right]^{1/3} \qquad (6)$$

Equation (6) is further simplified as

$$\hat{\nu}^3 - \hat{\nu}_t^2\hat{\nu} + (C-1) = 0 \qquad (7)$$

with a positive H function condition,

$$\hat{\nu}\hat{\nu}_t^2 - C \geq 0 \qquad (8)$$

The mixing length for wall bounded flow was taken from Martinelli and Yakhot[2]

$$l_1 = \frac{ky}{1 + \beta(y/\delta)} \qquad (9)$$

for transition prediction and from Kirtley[4] in turbulent flow region.

$$l_2 = C_\mu \delta \tanh\left(\frac{\kappa y}{C_\mu \delta}\right) \qquad (10)$$

The reasons for using these two length scales will be discussed in Section 3 and 4.

3. EVALUATION OF RNG LENGTH SCALE

The RNG length scale originating from Martinelli & Yakhot[2] [Equation (9)] has been analyzed using the Stock & Haase[5] analysis and compared with the Baldwin-Lomax[6] model.

By substituting Equation (9) into Equation (3), the provisional eddy viscosity, i.e., the maximum eddy viscosity from RNG model, can be written as

$$\nu_t = l_1^2 \left|\frac{\partial \bar{u}}{\partial y}\right| = \left(\frac{\kappa y}{1 + \beta(y/\delta)}\right)^2 \left|\frac{\partial \bar{u}}{\partial y}\right|$$

or

$$\nu_t = \frac{\kappa^2 (y/\delta)\delta}{[1 + \beta(y/\delta)]^2} y \left|\frac{\partial \bar{u}}{\partial y}\right| \qquad (11)$$

Stock and Haase[5] rewrote $y\left|\frac{\partial \bar{u}}{\partial y}\right|$ as function of y/δ using Coles velocity profile formulation:

$$y\left|\frac{\partial \bar{u}}{\partial y}\right| = \frac{U_e}{\kappa}\left[\pi|\sigma|\Pi\frac{y}{\delta}\sin(\pi\frac{y}{\delta}) + \sigma\right] \qquad (12)$$

The expression for $y\left|\frac{\partial \bar{u}}{\partial y}\right|$ can be approximated by taking its maximum.

$$Q = \frac{y}{\delta}\sin(\pi\frac{y}{\delta}), \quad \text{for } 0 < \frac{y}{\delta} < 1$$

where, the maximum value of Q can be obtained by Newton's method;

$$Q|_{max} \simeq 1.82 \quad \text{at} \quad \frac{y_{max}}{\delta} \simeq 0.646 \qquad (13)$$

Using these values, Equation (12) becomes

$$y\left|\frac{\partial \bar{u}}{\partial y}\right| \simeq \frac{U_e \sigma}{\kappa}[1.82\Pi + 1] \qquad (14)$$

The result of substituting Equation (14) into (11) is:

$$\nu_t \simeq \nu_{t,rng1} = \frac{(0.372)^2(0.646)\delta}{[1 + (4.44)(0.646)]^2}\frac{U_e \sigma}{\kappa}[1.82\Pi + 1]$$

or

$$\nu_{t,rng1} \simeq 0.006\, U_e \frac{\delta\sigma}{\kappa}[1.82\Pi + 1] \qquad (15)$$

The same procedure has been performed by Stock and Haase[5] for the Baldwin-Lomax[6] turbulence model's outer layer viscosity formulation yielding,

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$$\nu_{t,bl} \simeq 0.0173\, U_e \frac{\delta\sigma}{\kappa}[1.82\Pi + 1] \qquad (16)$$

Using Equations (15) and (16), the ratio of $\nu_{t,rng1}$ and $\nu_{t,bl}$ is:

$$\frac{\nu_{t,rng1}}{\nu_{t,bl}} = \frac{0.006}{0.0173} \simeq 0.35 \qquad (17)$$

From Equation (17), it is shown that the RNG viscosity predicted by the length scale of Equation (9) is about one third of that from the first Baldwin-Lomax outer model of Equation (16) that covers the near wall and attached flow regions. Consequently, it can be concluded that the length scale of Martinelli and Yakhot[2] predicts a much lower value of the eddy viscosity than the length scale of Baldwin-Lomax outer model. Also it has been shown that this length scale resulted in very low skin friction coefficients for flat plate and airfoil flows.

The length scale of Kirtley[4] [Equation (10)] has been introduced in the current RNG model in combination of that of Martinelli et al.[2]. The same analysis of Stock & Haase[5] can be performed again. Since the maximum length scale occurs for $|\tanh x|_{max} = 1$ in Equation (10), the maximum ν_t for the model is obtained from Equation (3).

$$\nu_{t,rng2} = (C_\mu \delta)^2 \left| \frac{\partial \bar{u}}{\partial y} \right| \qquad (18)$$

or

$$\nu_{t,rng2} = C_\mu^2 \delta \left(\frac{\delta}{y} \right) y \left| \frac{\partial \bar{u}}{\partial y} \right|$$

for provisional RNG viscosity. Again, using Equation (14) and corresponding y_{max}/δ value,

$$\nu_{t,rng2} \simeq C_\mu^2 \delta \frac{\delta}{y} U_e \frac{\sigma}{\kappa}[1.82\Pi + 1] \qquad (19)$$

or

$$\nu_{t,rng2} \simeq 0.0125\, U_e \frac{\delta\sigma}{K}[1.82\Pi + 1]$$

From Equations (16) and (19),

$$\frac{\nu_{t,rng2}}{\nu_{t,bl}} = \frac{0.0125}{0.0173} \simeq 0.72 \qquad (20)$$

From equation (20), the length scale of Kirtley[4] shows a more comparative magnitude of eddy viscosity than

the previous RNG length scale of Martinelli & Yakhot[2] [Figure 3]. Also, the results of skin friction for a flat plate and an airfoil using the length scale of Kirtley[4] are in better quantitative agreement with experimental data than the results obtained with the length scale of Martinelli & Yakhot[2]. However, one drawback in using the length scale of Kirtley[4] is that it predicts the laminar-turbulent transition point too early, especially for the flows with relatively low free stream turbulence intensities.

After the test of the RNG model using the length scales of Martinelli et al[2] (l_1) and Kirtley[4] (l_2) for flow over a flat plate and over a NACA 0012 airfoil, the previous analysis has confirmed that the length scale, l_1, from Equation (9) underpredicted the skin friction up to 30% but it accurately predicted the onset of transition from laminar to turbulent. Use of the length scale, l_2, from Equation (10) provides excellent agreement of skin friction in the fully turbulent region but predicts the transition point too early. Due to the sudden increase of the length scale near the wall ($0 < y/\delta < .2$) in the boundary layer, the solution sometimes diverges just after the model has been turned on.

4. IMPLEMENTATION OF RNG MODEL

Based on the analysis of the RNG length scales from the previous section, a new length scale which predicts the transition point accurately and also produces the correct turbulent skin friction coefficient is introduced.

Because the transition location $(y/\delta)_{tr}$ between inner and overlap layer inside a turbulent boundary layer is not known initially and it depends on many factors, such as initial free stream turbulence, Reynolds number, pressure gradient, surface curvature, and surface roughness, the actual generation of a single length scale may not be easily determined. Consequently, such an attempt was not made in the present implementation. However an RNG model is introduced, which makes a systematic use of the previous two length scales one at a time as follows:

The length scale, l_1 from Equation (9), is used first to find the accurate transition location and, once the positive H function condition is satisfied from Equation (8), the length scale, l_1 is replaced by l_2 from Equation (10) for the region where the RNG has been turned on.

From the solution of Equation (7) and accompanying condition Equation (8), the model is not turned on until the provisional eddy viscosity reaches the following

value:

$$\hat{\nu}_t = \frac{\nu_t}{\nu_0} = \sqrt{3} \left[\frac{(C-1)}{2} \right]^{1/3} \qquad (21)$$

where the provisional viscosity is calculated from the Equation (3) as follows:

$$\nu_t = l^2 \left| \frac{\partial \bar{u}}{\partial y} \right|$$

where

$$l = \begin{cases} l_1, & \text{for transition prediction;} \\ l_2, & \text{for turbulent RNG viscosity evaluation.} \end{cases}$$

The above implementation of two length scales can be depicted only if one new length scale were used as shown in Figure 4 where the new length scale follows the l_1 for $y/\delta \le (y/\delta)_{tr}$ and l_2 for $y/\delta > (y/\delta)_{tr}$.

In the actual implementation, unlike the Baldwin-Lomax model where the turbulent viscosity is calculated for the entire computational domain, the following algorithm was used in order to avoid unnecessary computation for turbulent viscosity in the laminar region:

Since the transition criterion for ν_t is known in Equation (21), for the laminar region, only the calculation of Equation (3) has been necessary and the evaluation of the RNG viscosity through the Equation (7) can be bypassed with the solution of $\hat{\nu} = 1$, which is the laminar viscosity value. Once the transition has been triggered by the condition in Equation (8) and ν_t reaches the critical value in Equation (21), evaluation of the RNG viscosity via Equation (7) is performed with ν_t calculated using l_2.

With its non-iterative evaluation scheme of the RNG viscosity from the cubic equation, [Equation (7)], the above length scale implementation shows much improved computational results relative to the previous one using either one of the above mixing lengths.

5. COMPUTATIONAL RESULTS

The Reynolds-averaged Navier-Stokes equations (continuity, momentum, and energy equations) can be expressed in the following conservation law form:

$$\frac{\partial \mathbf{Q}}{\partial t} + \frac{\partial \mathbf{F_j}}{\partial x_j} = \frac{\partial \mathbf{G_j}}{\partial x_j}$$

where $\mathbf{Q}$ is a vector containing the conservation variables,

$$\mathbf{Q} = \begin{bmatrix} \rho \\ \rho u_i \\ E \end{bmatrix}$$

where $\mathbf{F_j}$ vectors represent the invicid flux vectors,

$$\mathbf{F_j} = \begin{bmatrix} \rho u_j \\ \rho u_i u_j + p \delta_{ij} \\ (E + p) u_j \end{bmatrix}$$

and the viscous flux vectors $\mathbf{G_j}$ are

$$\mathbf{G_j} = \begin{bmatrix} 0 \\ \tau_{ij} \\ u_k \tau_{jk} - q_j \end{bmatrix}$$

The eddy viscosity concept and the algebraic forms of the Renormalization Group turbulence model are used for turbulence closure. A simple algebraic eddy viscosity model based on the Renormalization Group turbulence model is used for closure of the compressible Reynolds averaged equations. Two turbulent boundary layer flows using the RNG turbulence model with the mixing length proposed earlier were solved together with the Navier-Stokes equations for compressible flows in full conservation form. The numerical method is a finite difference discretization with the Beam and Warming[7] approximate factorization algorithm to integrate in time to a steady state. The calculations were performed on a Cray YMP computer at NASA Lewis Research Center.

5.1 Flow over a flat plate

Experimental data for the flow over a flat plate with a zero pressure gradient can be found in Sohn & Reshotko[8]. The Mach number was 0.2 and the Reynolds number based on plate length was approximately 2.3×10^7 with a physical dimension of 16 feet. The corresponding computational domain, covering only the upper half of the plate, starts from the free stream 1 foot ahead of the leading edge and extends to the end of the plate in the streamwise direction. The domain extends 3 feet high from the wall in the normal direction. A grid of 111×81 points was used with about one third of these located within $y^+ \le 1000$. About 15 grid nodes were put within $y^+ \le 50$ in the normal direction. The streamwise grid was densely packed near

the leading edge for an accurate prediction of the large gradients in the vicinity of the wall and the leading edge [Figure 5].

In order to check the grid-independence and also obtain a more accurate prediction of the transition location, the calculation had been repeated with a finer grid of 221 × 161 points. No variation of transition location had been obtained. The converged solution was obtained after approximately 10,000 iterations.

As shown in Figure 6, the transition can be located where the non-dimensional boundary layer thickness along the streamwise direction suddenly increases from a constant value. In the figure, the present computation clearly shows similar trend and agrees qualitatively with experimental data of Hanson[9]. The computational results for the skin friction coefficient and the shape factor are compared with experimental data. Figure 7 shows the local skin friction coefficient and Figure 8 shows the shape factor distribution. The solid line represents the computational results using the present RNG model, and the symbols represent the experimental data under different free stream turbulence intensities. The analytic results for laminar flow and a semi-empirical correlation for turbulent flow are also plotted. The results are in good agreement with the experiment, and it is shown that the present RNG model, without much dependence on empiricism, can accurately mimic the detailed phenomena of laminar-turbulent transition. Figure 9 shows the turbulent boundary layer velocity profile near the end of the plate showing excellent agreement with a correlation of Musker[10] in both inner and outer layers.

Setting up a convergence criteria for RNG simulation prior to computation is not straightforward since, as plotted in Figure 10, the final residual at convergence is not the minimum value throughout the computation due to a residual jump after the RNG model has turned on. In the figure, two curves are depicted from two simulations using different time step control; namely, time accurate simulation with constant time-step size (Δt=5) and local time stepping simulation with variable time-step size (maximum time-step size allowed during the run, Δt_{max}, is set to 20). The same skin friction results from the two cases were obtained after 15000 iterations for Δt=5 and 6000 iterations for Δt_{max}=20. The irregular convergence pattern after the model turned on for Δt_{max}=20 is due to larger time step and/or the local time stepping function of current code which also started after RNG has turned on. Meanwhile, the time accurate execution shows more regular pattern. In both cases, the sudden jumps in the residual are due to the jump discontinuity of eddy viscosity in RNG formulation. It should be noted, however, that the local disturbance due to switching on the model at transition region does not deteriorate other regions and overall computation remains stable.

5.2 Control of transition

The present RNG model uses the analytic relation of the δ/y_{max} in its length scale when determining the provisional eddy viscosity. It has been observed that, by increasing or decreasing δ/y_{max} from its standard value (=1.548), the change of the transition location can be simulated numerically as if the flow is in a different environment which affects the transition. The two computational results plotted in Figure 11 and 12 as dashed and dotted lines are RNG results obtained by using different boundary layer thickness parameters, δ/y_{max}. The results obtained by a 25% increase and decrease of δ/y_{max} show good qualitative agreement with experimental data obtained with higher and lower free stream turbulence intensities, respectively. This confirms that, as mentioned earlier, this parameter can be used to predict the transition location under different flow environments which affect the transition location.

5.3 Flow over a NACA 0012 airfoil

Another calculation was performed for the flow over a NACA 0012 airfoil with zero lift in order to test the present turbulence model for a flow with a pressure gradient. The experimental data considered here can be found in Becker[11]. The air speed was 230 mph and the Reynolds number based on the airfoil chord was approximately 1.0×10^7 with a chord length of 5 feet and a maximum airfoil thickness of 12% chord. The corresponding computational domain covers the entire airfoil starting from the free stream at one chord length ahead of the leading edge and extending to 5 chord lengths in the streamwise direction. The domain extends one chord length from the wall in the cross-stream direction. A grid consisting of 358 × 100 points was used, with about one-third of these located within $y^+ \leq 1000$ in cross-stream grid nodes. The streamwise grid has also been densely packed near the leading edge for an accurate prediction of the large gradients in that region [Figure 13].

The grid-independence was checked by refining the grid in both directions. Two hundred more grid points were added in the streamwise direction. In the direction normal to the stream, more grid nodes have been included near the wall which reduces the y^+ value to below one. The converged solution was obtained after approximately 6,000 iterations.

The computational results for the skin friction co-

efficient and boundary layer velocity profiles were compared with experimental data. Figure 14 shows the local skin friction coefficient, and Figure 15a through Figure 15c show the boundary layer velocity profiles at different streamwise locations. The solid line represents the computational results using the present RNG model, the symbols represent the experimental data. Since the experiment was performed in a high speed wind tunnel with very low free stream turbulence ($\simeq$ 0.01%), the use of the standard value of δ/y_{max} [Equation (13)] resulted in the transition location being predicted at 4% chord while the experimental value is approximately 15% chord. This is consistent with the fact that, from flat plate results, the standard value of δ/y_{max} predicts the experimental transition location with relatively high turbulence intensity which usually triggers early transition (see Schlichting[12]). The transition location has been delayed by decreasing the δ/y_{max} value following the earlier analysis, and the transition location of 15% chord was obtained with δ/y_{max} 40% lower than the standard value.

In Figure 16, the pressure coefficient has been compared with experimental data for the upper surface of the airfoil. The results indicate good agreement with the experimental static pressure distribution along the streamwise surface.

6. CONCLUSIONS

The renormalization group algebraic turbulence model was introduced in order to test its performance on boundary layer transition simulation. The new implementation method enables the use of two different length scales in systematic way in order to predict transition location better and also produce accurate turbulent flow quantities. The followings are detailed conclusions of the present analysis:

1) On the basis of the boundary-layer results, it was shown that dropping one solution branch of the cubic equation derived from the RNG theory does not affect the performance of the model nor the convergence of the Navier-Stokes calculation.

2) The analytic study and computational results show that the RNG length scale introduced by Martinelli and Yakhot[2] provides excellent predictions of the transition location, but it underpredicts the magnitude of eddy viscosity resulting in a lower skin friction coefficient.

3) The RNG length scale introduced by Kirtley[4], provided excellent predictions of fully turbulent quantities, but indicated an early transition location and rapid

transition to fully turbulent flow which can cause numerical instability.

4) A length scale which uses the two previous RNG length scales in a systematic way to preserve the merits of the two scales is introduced. This new length scale improved the model's performance in predicting transition as well as the turbulent quantities as shown through a flat plate and an airfoil calculation.

5) For separated flows or strong adverse pressure gradient flows, the analytic study of the RNG length scale shows the need to modify either the RNG length scales, or the method to evaluate the provisional eddy viscosity, in order to avoid too large of an eddy viscosity value.

6) The present model's capability of predicting the transition location under different flow conditions has been introduced and delivers satisfactory results for flows with different free stream turbulence intensities.

7) The fact that the present RNG model does contain less experimental constants or adjustments, and yet gives accurate transition predictions as well as fully turbulent predictions with its simple algebraic formulation makes the model general, realistic, cost-efficient and comparable to the other low-Reynolds number two equation turbulence closure models.

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APPENDIX

Implementation to PARC code

The renormalization group theory (RNG) turbulence model in the present paper has been successfully implemented into the two- and three-dimensional PARC code with an additional subroutine RGTURBW and one function subroutine EDR. The user can select the RNG turbulence model by setting the namelist input parameter IMUTUR=4. Also in order to correctly predict the transition location under different flow conditions, such as free stream turbulence or surface roughness etc., the user can set a new namelist input parameter D2OYMX within ± 40% of its default value (=1.548) to move the transition point. The default value of D2OYMX can simulate a turbulence intensity of about 2% for the flow over a flat plate with zero pressure gradient. The user can set a lower value than 1.548 for D2OYMX to simulate the flow with a lower free stream turbulence.
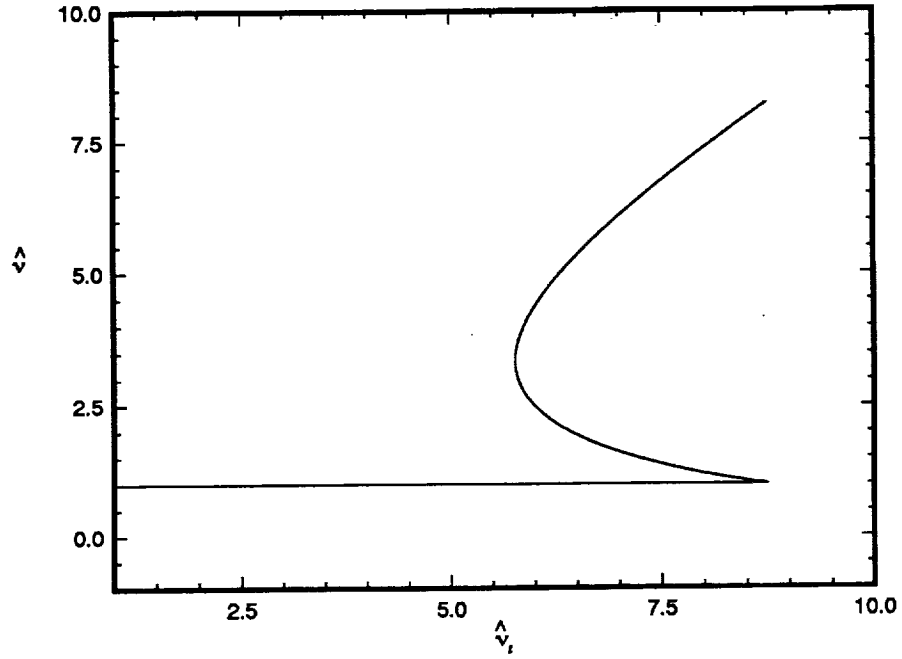
Figure 1. RNG viscosity, $\hat{v}$ as function of provisional viscosity, $\hat{v}_t$ via cubic equation for C=75.
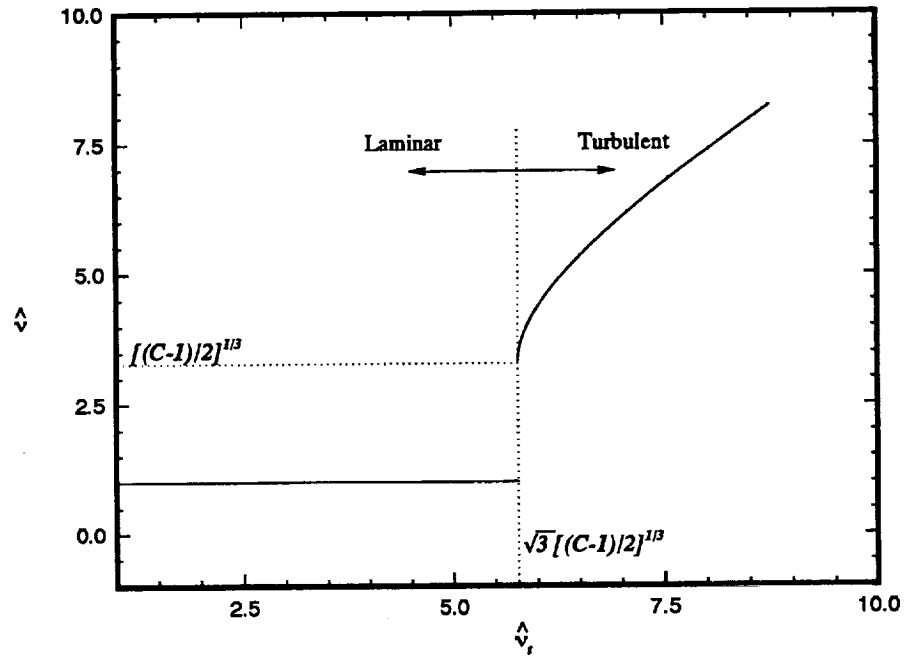
Figure 2. Modified RNG viscosity, $\hat{v}$ as function of provisional viscosity, $\hat{v}_t$ via cubic equation for C=75.
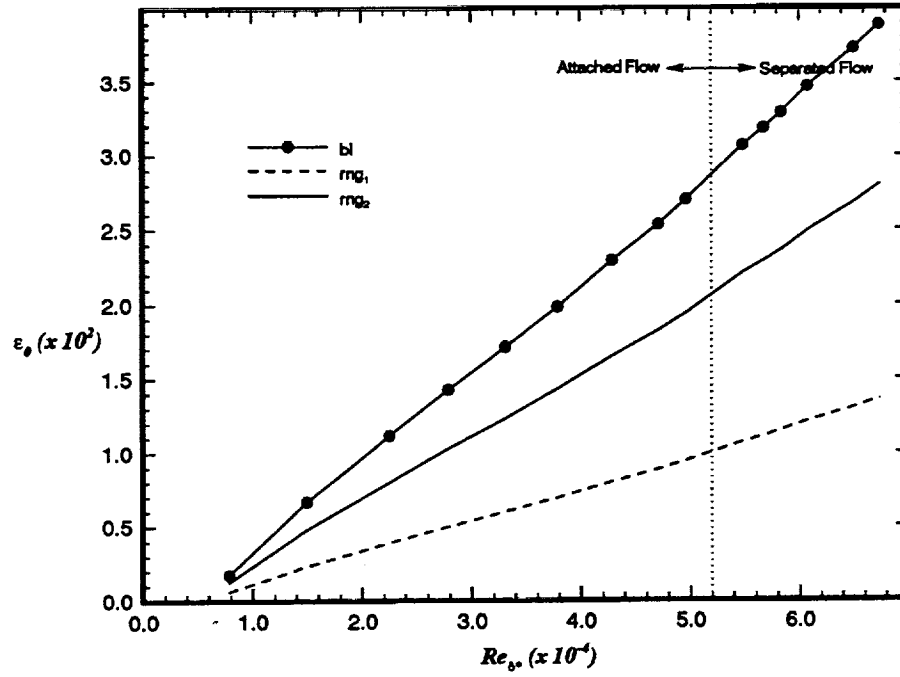
The labels in Figure 2: Laminar, Turbulent, $[(C-1)/2]^{1/3}$, $\sqrt{3}[(C-1)/2]^{1/3}$

Figure 3. Comparison of RNG viscositys with outer-eddy viscosities of Baldwin-Lomax turbulence model.
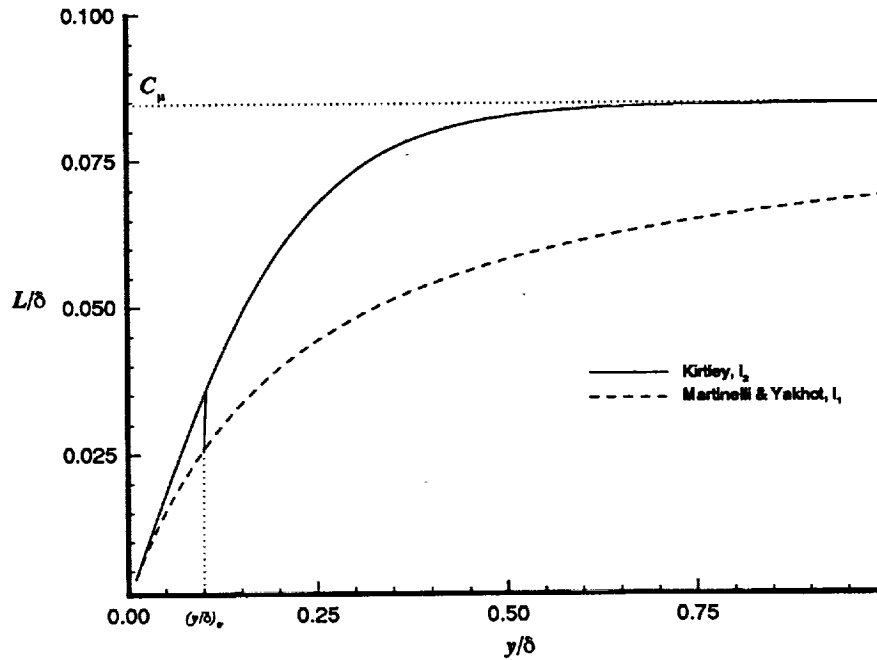
Figure 4. Length scales used in the present model and systematic transition from one length scale to another inside a turbulent boundary layer.
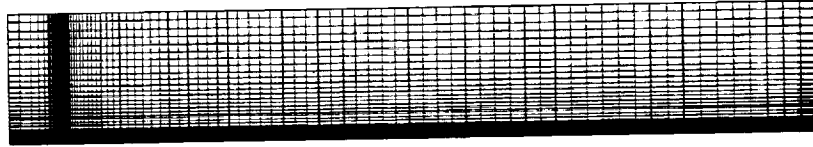
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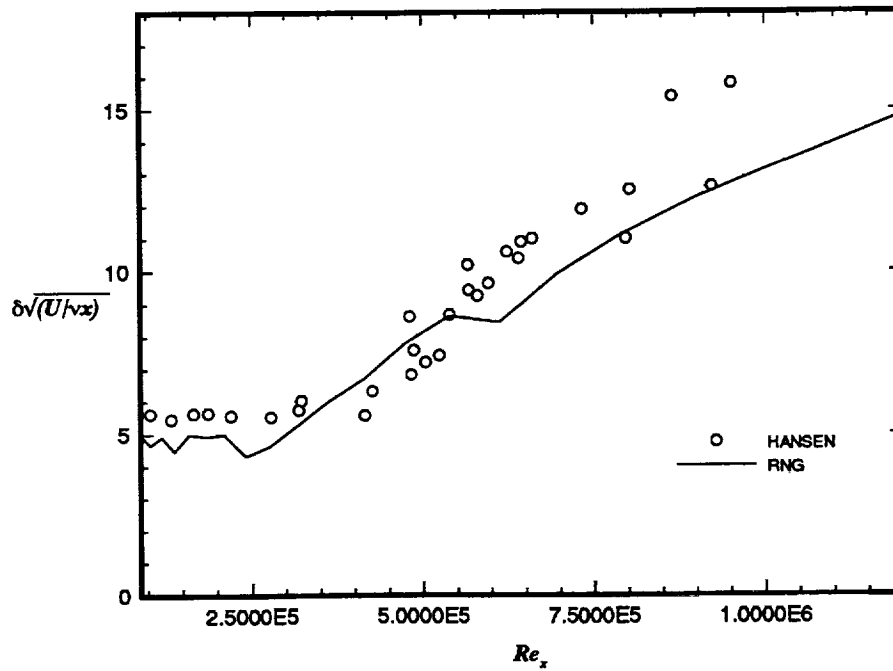
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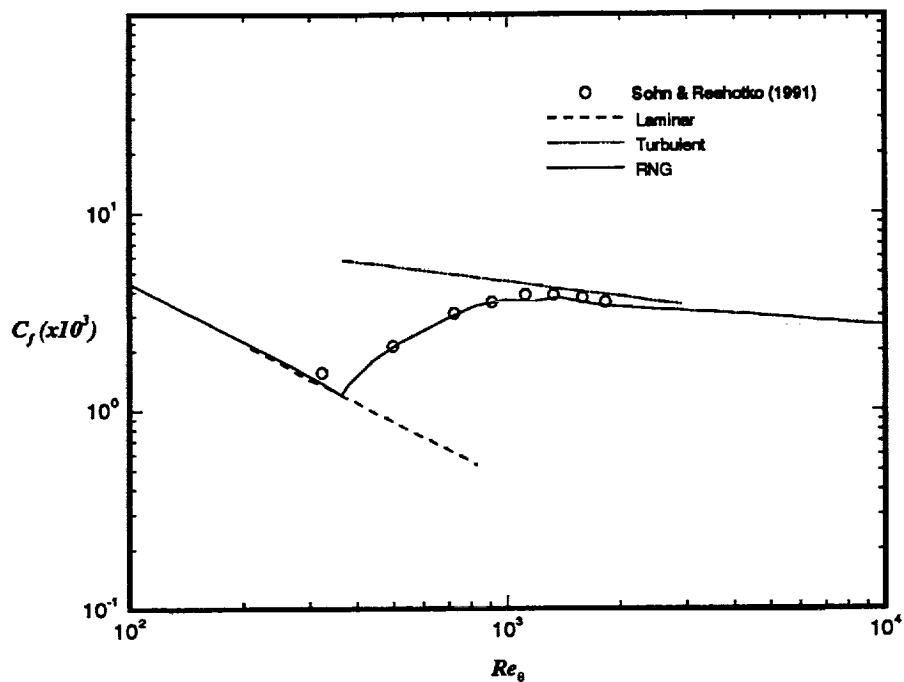
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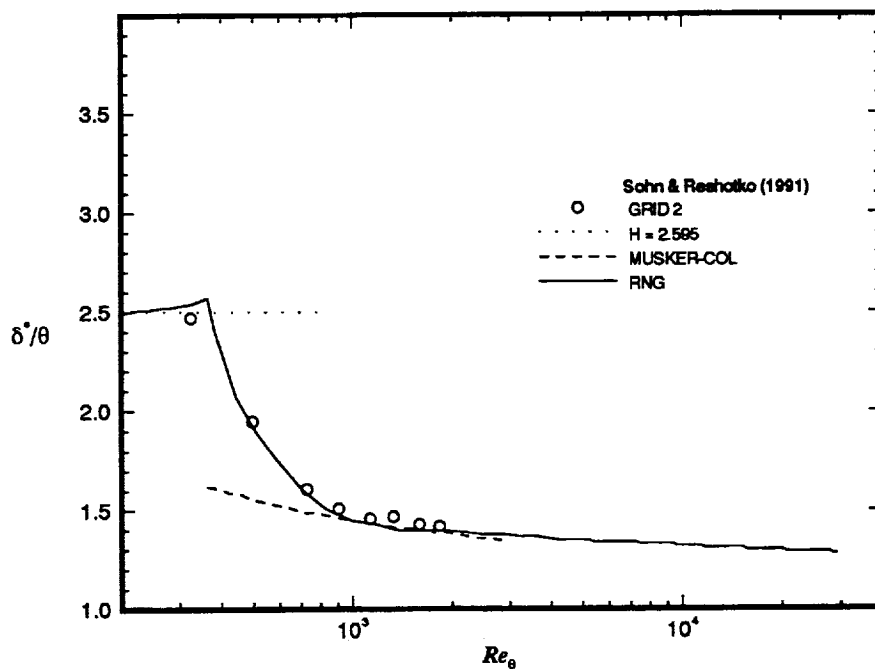
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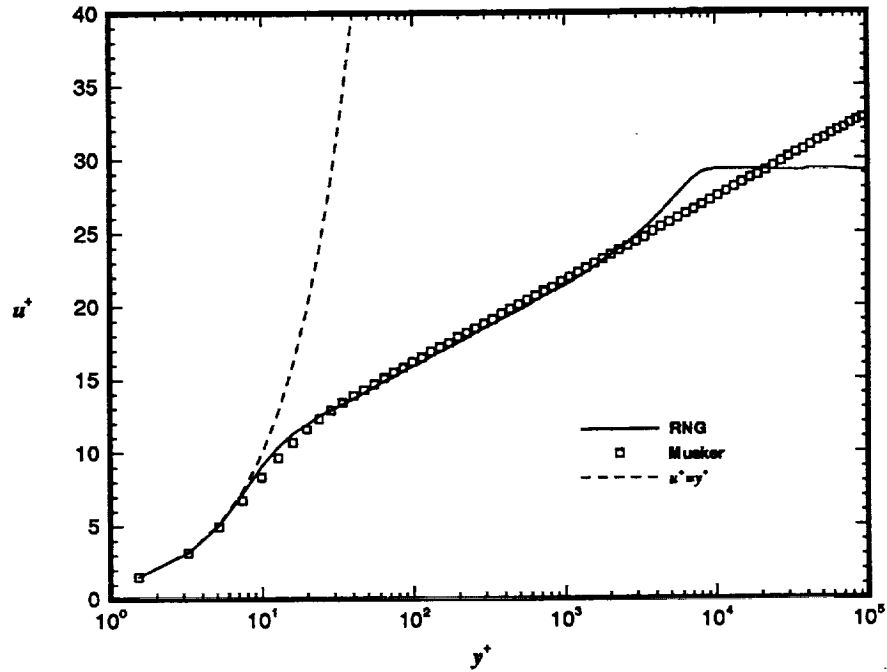
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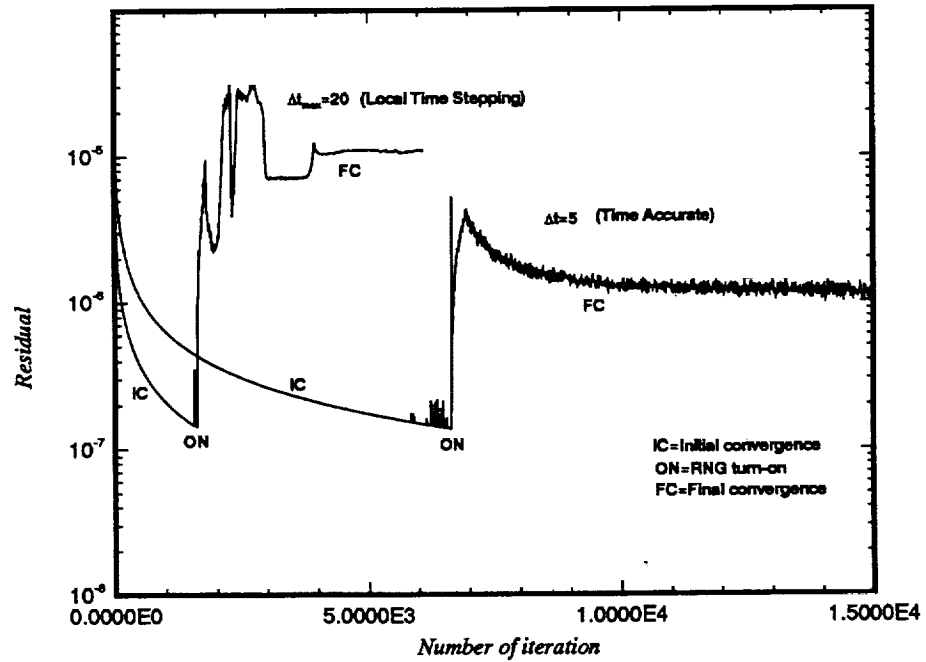
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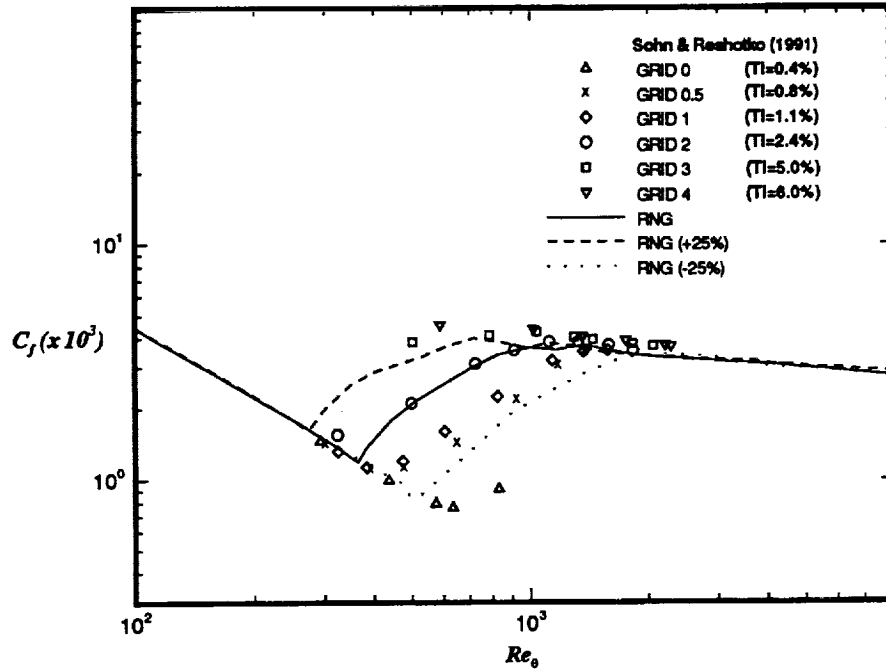
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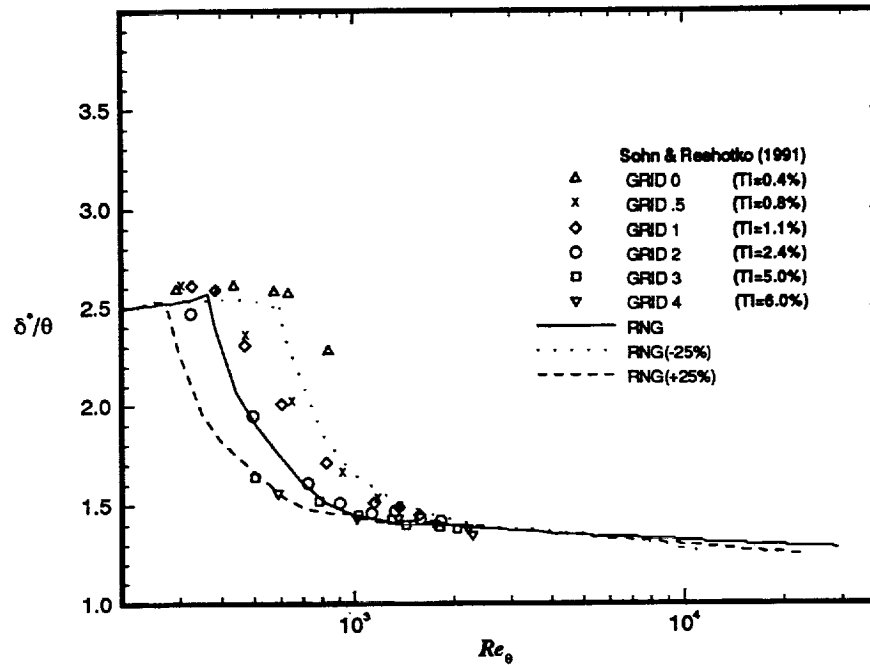
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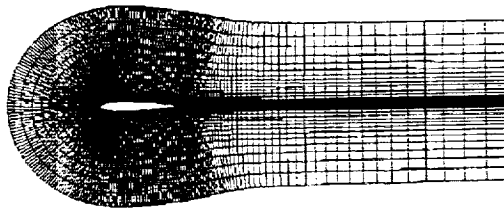
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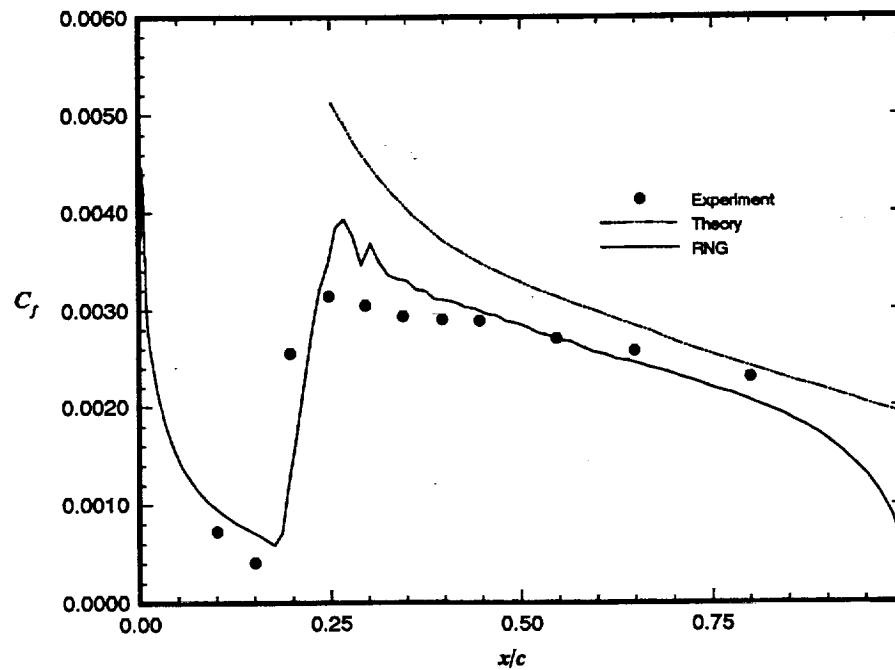
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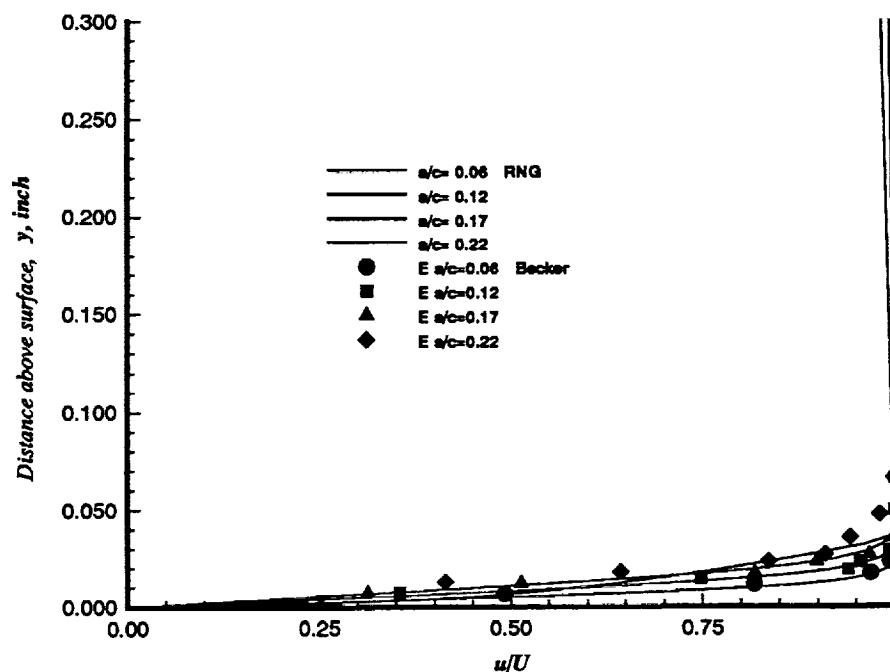
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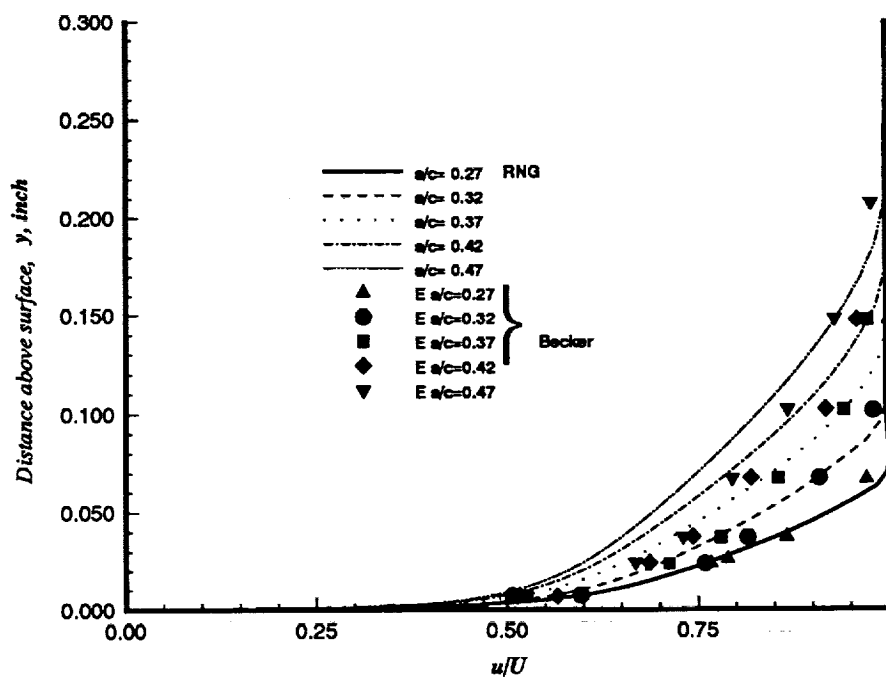
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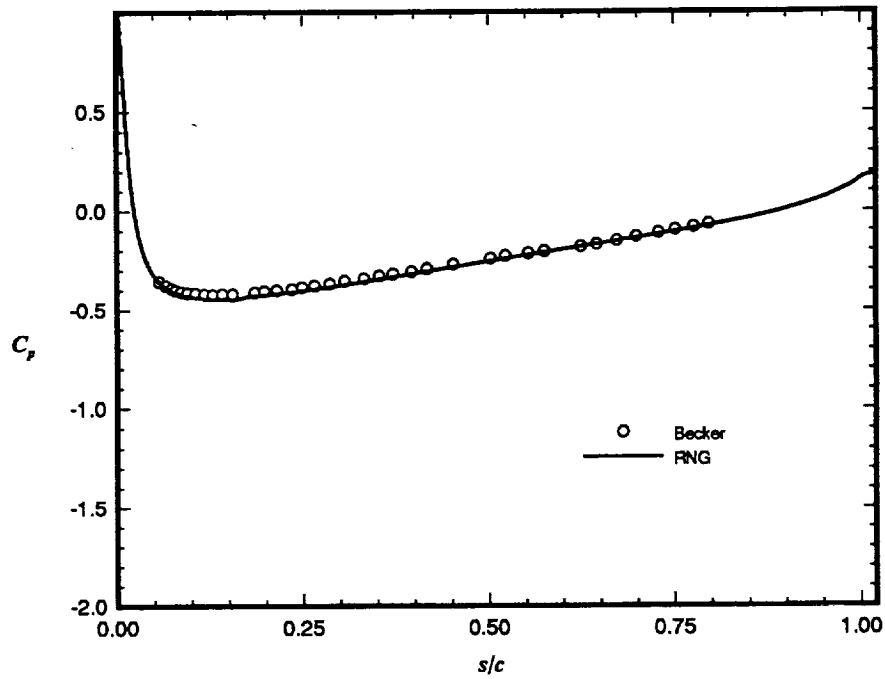
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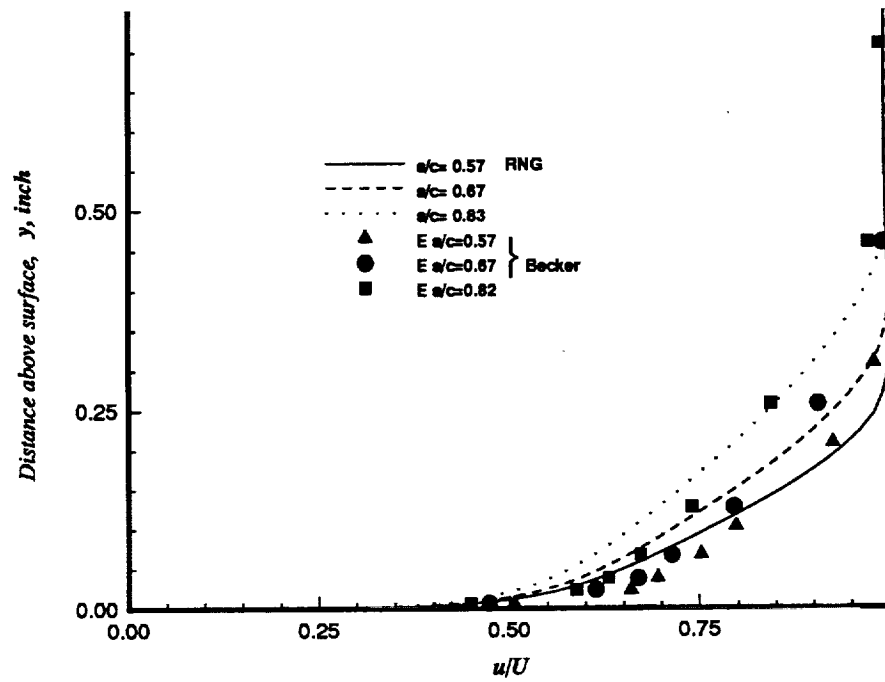
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13. ABSTRACT (Maximum 200 words)

The RNG-based algebraic turbulence model, with a new method of solving the cubic equation and applying new length scales, has been introduced. An analysis has been made of the RNG length scale which was previously reported and the resulting eddy viscosity has been compared with those from other algebraic turbulence models. Subsequently, a new length scale is introduced which actually uses the two previous RNG length scales in a systematic way to improve the model performance. The performance of the present RNG model has been demonstrated by simulating the boundary layer flow over a flat plate and the flow over an airfoil.

14. SUBJECT TERMS

Renormalization group (RNG); Computational fluid dynamics (CFD); Turbulence model transition; Mixing length; Flat plate; Airfoil

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